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On Their Own:

Student Response to Open-Ended Tests in Mathematics

MASSACHUSETTS DEPARTMENT OF EDUCATION

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On Their Own: Student Response to Open-Ended Tests in Math

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Dear Educator:

The four booklets in this series discuss the reading, mathematics, science and social studies results of the 1988 Massachusetts Educational Assessment Program. They represent one of the many efforts of the Department of Education to help schools carry out their educational mission more effectively. In this case, they provide models for student evaluation within the classroom, as well as describing students' progress in understanding.

The title of this series, **On Their Own**, suggests an important aim of education: the ability of students to act as independent, rational thinkers. The questions described in these booklets demand that ability. They demand active intelligence as students are required to relate what they know to new and challenging situations.

In addition to describing students' understanding, these booklets carry a message about the evaluation that goes on in the classrooms. The message is that the short objective tests of facts or procedures, standard fare in most classrooms, are too slight a vehicle to convey the true purpose of evaluation.

In the first place, effective student evaluation is an important component of effective teaching. Research has described the complex thinking that underlies students' errors and misconceptions. Unless teachers take the time to discover for themselves how students understand a subject, they will be unable to adjust their teaching in appropriate ways. This kind of evaluation, involving student discussion and explanation, should be a continuous and constant part of every classroom.

Secondly, evaluation can, and does, affect students' learning. Not only does it signal for the student the content areas that teachers consider important, it gives a message about the kind of thinking that is considered valuable. When testing is limited to short objective questions, requiring a single answer, the message given is that facts are what really count. When questions encourage students to think, to grapple with the material and to consolidate different aspects of learning, the message is much different. Such questions indicate to students that it is the quality of thought that is important, not the correctness of the answer itself. The possibility of different answers opens the door for discussion, argumentation, and intellectual excitement in our schools. This is the message that we want to convey to our students.

We hope that you will study the material included in this series and incorporate the ideas presented in your own classrooms.

Sincerely yours,

Harold Raynolds Jr.

Harold Raynolds Jr.
Commissioner of Education

Acknowledgments

This report would not have been possible without a major contribution from members of the Mathematics Advisory Committee. These teachers and mathematics coordinators analyzed the responses for each question, read and scored the scripts, and interpreted the results with reference to both student achievement and school instruction. It was a major project, which they accomplished with competency, efficiency, grace, and goodwill. If you find this book at all useful, it is they who should be thanked.

Members of the Mathematics Advisory Committee who contributed to making this booklet possible are:

Thomas Carroll	**Chelmsford Public Schools**
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Elizabeth Badger

Table of Contents

Foreword

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Critics claim that there are many problems with current assessment procedures. Commonly used tests reflect a fragmented view of mathematics rather than the view of mathematics as an integrated whole and they predominantly consist of items requiring factual knowledge or procedural skills rather than items that assess students' abilities to think critically, to reason, to solve problems, to interpret, and to apply ideas in creative ways. With their focus on solutions rather than on strategies or processes, the current tests reinforce in students, teachers, and the public the narrow image of mathematics as a subject with unique correct answers.

Critics of current mathematics assessment procedures are consistent in what they proposed to replace the current procedures. It is essential that testing be reorganized to parallel the new goals of the curriculum: the development of mathematical understanding, the interpretation of mathematical events, and the application of mathematical procedures. Because the emphasis should be not only on solutions but also on processes and strategies, tests should include open-ended or free-response items, that is, items for which the examinee has to create a response rather than select a response from a list.

This report presents information about Massachusetts initial use of open-ended questions in their biennial assessment of mathematics at grades four, eight, and twelve. The summary of results provided here make the importance of including such items in any assessment apparent.

Notes

Introduction

In the spring of 1988, the Massachusetts Department of Education administered its second biennial assessment. It tested all eligible fourth, eighth, and twelfth grade students in four content areas – reading, mathematics, science, and social studies. Although the large majority of the over 3000 items were given in multiple-choice format, some of the items were open-ended, requiring students to answer in written form. These open-ended questions appeared in one form of the tests at each grade level. Consequently, one-twelfth of the fourth grade students, one-sixteenth of the eighth grade students, and one-twentieth of the twelfth grade students received a test form that contained some open-ended questions. However, they did not all receive the same questions. The ten or so questions in each subject area were distributed in such as way that, while we were able to receive a sufficiently large number of responses to each question to report reliably, we were not able to cover the many types of thinking that each subject required.

Members of our Advisory Committees, composed of teachers throughout the state, reviewed and categorized all the answers obtained. Their comments and instructional suggestions are reflected in this report.

There are three reasons for our decision to include open-ended questions in our assessment of student performance. The first is the value of the information obtained. While multiple-choice items are efficient, easy to score, and objective, they are a weak measure of how students actually think. Nor can they measure students' ability to generate solutions or students' approach to those ill-structured problems that are most familiar in everyday life. Including these types of questions on the assessment results in a more valid estimation of student achievement than we would have obtained had we limited assessment to multiple-choice items.

Our second purpose in including these open-ended questions was our belief in their intrinsic value: they reflect the kind of thinking that education is all about. Too often educators pay lip service to the need for active learning but teach and test students in ways that demand passivity. By their actions, schools say to students, "We are not interested in *your* response; we are only interested in the *correct* response." This report of the open-ended testing

shows how students respond when they are challenged to conceptualize a problem as well as deal with it.

Finally, we hope that such testing will act as a model for classroom testing. We have given the questions themselves, as well as the state-wide results, in order for teachers to try them out in their own classes and, if they desire, to compare the results they obtain with the state norm. We wish to show that this type of testing yields important information about students' understanding of concepts and procedures, their ability to apply their learning to new situations, and their need for further instruction.

Although the results in each subject area are treated separately, the underlying thought processes which we report on are similar. They reflect an approach to thinking that stresses engagement and critical evaluation. Beyond this, however, we look at how students function in various learning contexts which require the understanding of specific concepts.

A New Vision of School Mathematics

During the last two years, it has been generally conceded that there is something wrong about the way mathematics is taught and learned in this country. When the National Assessment for Educational Progress issued its **Mathematics Report Card**, students received poor marks across the board.[1] It was reported that virtually no 9-year-olds, and only 16 percent of 13-year-olds and 51 percent of 17-year-olds could deal with moderately complex mathematical procedures and reasoning, e.g., finding the area of a rectangle. Only 6 percent of 17 year olds could use more complex reasoning or algebra to solve problems.

Later in the year, an international study of mathematics and science achievement among 13-year-olds confirmed these results.[2] United States students obtained the lowest mathematics score of any participating group. In one category, the use of intermediate-level skills to solve two-step problems, there

1 Dossey, J. A., et al, *The Mathematics Report Card.* Princeton, N.J.: Educational Testing Service, 1988.

2 Lapointe, A.F., Mead, N., and Phillips, G. *A World of Differences. An International Assessment of Mathematics and Science.* Princeton, N.J.: Educational Testing Service, 1988.

was a 38 percent difference between the achievement level of 13-year-olds in this country and in Korea, the high scoring country. Only 9 percent of our students showed an understanding of measurement and geometry concepts, in contrast to 40 percent of Korean students.

The following year, the National Research Council and the National Council of Teachers of Mathematics (NCTM) suggested remedies. They looked to the future with two publications that challenged the way that mathematics is conceived of and taught in the schools and gave their vision for a more meaningful mathematics education.[3]

A key point made by both organizations is that mathematics is and always has been an evolving intellectual area that connects the theoretical and practical world. From the time that Kepler anticipated the concept of integral calculus by trying to calculate the best shape for Austrian wine casks to the present concern of mathematicians in trying to define the structural pattern of random events in natural phenomena, mathematics has consisted of both abstraction and application, constantly crossing between the worlds of ideas and practicality. This is something that few students realize as they march, and often stumble, through the grind of arithmetic calculations.

The National Council of Teachers of Mathematics believes that appreciation of what mathematics really is should permeate its teaching. One's first impression when reading through the NCTM's **Curriculum and Evaluation Standards** is the variety of mathematical topics that are covered. One's second impression is that many of the same ideas and concepts reappear in different forms. Mathematics is a connected body of thought. It is connected, not only with other subjects—science, social studies, as well as art and music—but as a subject within itself. An underlying structure unites the various mathematical topics, while the same phenomena can be represented visually, algebraically, statistically. An appreciation of that basic connectedness (structure) is what the authors of the **Standards** call "mathematical power."

How students can attain that power is what the **Standards** are about. They describe the basic concepts and skills that are appropriately taught throughout the school years, justify their mathematical importance, and give

3 National Research Council, *Everybody Counts.* Washington, D.C.: National Academy Press, 1989. National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: NCTM, 1989.

guidance to teachers on how they should be presented. The vision that they describe emphasizes the logical consistency of mathematics.

The ideas that underlie these mathematical goals are very similar to those that underlie the reform in other subjects—the importance of prior knowledge and preconceptions,those ideas and concepts that students bring to the classroom experience; the need for active learning, whereby students make ideas their own; the explicit reference to real-life problems; and the importance of reasoning and communication. However, in contrast to other subject areas, this vision could be seen as revolutionary. It is certainly not the picture of mathematics as it is taught in the majority of schools today.

We use a single incident to illustrate our point. During our pilot testing of the open-ended items, a twelfth grade student looked at them with a sense of outrage and muttered, "These aren't math!" Putting his head on the desk, he refused to go on. This student was clearly insulted by being confronted with something that was unfamiliar, requiring him to apply what he had studied to a context that was different from his daily routine of textbook problems. He was unprepared even to try. And yet, few of the problems contained in the open-ended form of the assessment were "difficult" problems. Many could be answered by common sense or by applying some fundamental mathematical concepts. Their difficulty lay, not in the knowledge that they required, but in their unfamiliarity and their demand for understanding rather than procedure *per se*. In this regard, they are similar to the demands of those so-called mathematical problems of everyday life. Mainly, they demand a flexibility, an ability to see the underlying mathematics and to relate learned knowledge to a new situation. Unless students are exposed to these kinds of problems they will continue to see mathematics as something useless, that is carried out in a 40-minute class, with no relevance to the real world outside the school boundaries.

The set of open-ended questions which we present in this booklet include some fundamental ideas of mathematics. They are not designed as models for the kinds of problems that required extended investigation. Although extended problems should be part of everyone's school experience, they are not appropriate for the limited time period allowed for the assessment. The problems presented here are intended as models for teaching, as well as for evaluation. These are classroom exercises that are appropriate for all students. In that context they serve a double role. They allow students to begin to think and talk about mathematics, to share their ideas and, perhaps, to become excited. Each could be a starting point for class or individual discussion. They also allow teachers to gain insight into how their students think

about mathematics. When practical, each of the questions is also accompanied in the Appendix by a chart giving the percentages of responses throughout the sample. These are presented for the convenience of teachers who might wish to compare the performance of their own students with those across the state. Mathematics teachers should also consult a companion publication, **On Their Own: Student Response to Open-Ended Tests in Science**, for problems involving data reduction and analysis.

Notes

A Question of Answers

The open-ended items that were presented in the assessment covered three major areas in school mathematics: Patterns and Relationships, Geometry and Measurement, Numerical and Statistical Concepts.

Patterns and Relationships

Experience in recognizing patterns and relationships is not an "enrichment activity." To a much greater degree than arithmetic calculations, it represents the real work of mathematics, with two important benefits for students. One is affective. If we want students to appreciate mathematics, to see it as a reasonable activity that they can control for their own purposes, then we cannot just give them a set of rules to follow. They must be able to act like mathematicians, discovering for themselves the numerical and geometric structures that are the essence of mathematics. Discovering patterns is not only satisfying in itself, it gives students a sense of confidence in the reasonableness of mathematics.

Secondly, work with numerical and graphical patterns lay the groundwork for understanding functions and the relationship between variables. Without an intuitive feeling for what the symbols of algebra represent, algebraic manipulations remain just that – manipulations.

Numerical Patterns and Relationships

Grade Four

Fourth grade students' ability to recognize structure in numerical patterns was tested in the following question, which was also asked on the multiple-choice section of the test, without the requirement that students explain their method.

> When numbers are put in a number machine, different numbers come out.
>
> If a 3 goes in, a 5 comes out.
> If a 4 goes in, a 7 comes out.
> If a 5 goes in, a 9 comes out.
>
> If an 8 goes in, what number comes out?
>
> Explain in words how you came up with your answer.

The table below shows how the responses to the two versions of the question differed.

Number produced	Percentage of responses: open-ended	multiple-choice
15 *	39	28
12	15	50
14	5	18
23	-	4
13	19	-
11	4	-
other	16	-
blank	2	0

The students who were given the open-ended version of the question were more successful than those who answered the multiple-choice version. Perhaps the requirement that they explain their method prompted them to think more carefully about the relationship. In any case, their explanations gave us some insight into how students arrived at incorrect responses on both versions of the question. For example, half the students incorrectly chose 12 as the answer when presented with pre-selected options, while 15 percent gave it in

response to the open-ended. From the explanations given, it was apparent that these students focused exclusively on the final pair of numbers (5 and 9), without considering all the sets of numbers. Noting a difference of four, they generalized: **I got 12 because 8 plus 4 is 12** or **If you put 5 in, a 9 comes out. So if you put 8 in a 12 must come out because of 6..7..8..9 and 9..10..11..12. 12 is the last answer, so it must come out.**

Although 14 was a popular choice when presented as an option in multiple-choice, few students volunteered it when they had no options to choose from. Instead, many (20 percent) gave the answer as 13. In general, this seemed to be the result of carelessness rather than a misunderstanding of the relationship between the two sets of numbers. For example, many students recognized the functional relationship but ignored or miscounted the missing elements in the series (6 and 7). A typical explanation for this type of error was: **The first one adds 2 numbers, and the second one adds 3, and the third one adds 4, and so on. Number 8, you have to add 5 to it and it's a 13.**

A large number (16 percent) recognized only very general relationships or guessed: **Because all the numbers that go in are even, and when they come back out they are odd.**

As well as lending insight into why students chose the incorrect answer, the requirement for explanation also gave an indication of students' reasoning when they chose the correct solution. It cast doubt on how well some students understood the relationship involved. For example, of those who gave the correct answer of 15, only a third were able to spot some kind of functional relationship which they were able to express in words.

> You double the number you have and subtract one.
>
> I looked at all the figures and subtracted the number that went in from the number that came out. I found out that the answer was 1 less than the number that went in. Then I took 1 off of 8, which was 7. Then I said 8 + 7 = 15.

Another third focused only on the second set of numbers, treating it as a sequence: **Each number that comes out is 2 away from each other.** A final third who gave the correct answer could not explain their reasoning: **I found out the pattern and completed it.**

Grade Eight

At eighth grade, we asked students for an algebraic as well as a verbal explanation of the relationship between two sets of variables.

Students were given the following question:

Sample:

x	y
0	1
1	2
3	☐
4	5
6	7

The missing number:

What you do to x to get y:

The equation:

Now try this one:

x	y
0	1
2	5
4	☐
7	15
9	19

The missing number:

What you do to x to get y:

The equation

As in the previous question, the correctness of the solution gave only slight indication that students understood the relationship involved. (Results are given in Appendix.)

Although approximately half the students gave the correct answer of 9, their subsequent explanations showed that far fewer recognized the function that existed between the two sets of numbers. A substantial number (the 10 percent who answered "$y = x+5$") focused on the specific case. They may have been misled by the example given ($y = x + 1$), in which the equation both described the missing number and the set of relationships. However, they most certainly did not understand the question as asking for the function that

relates the two **sets** of numbers. Another 13 percent, while giving the correct number, were unable to supply an adequate explanation or an equation.

The remaining half of the students were obviously baffled by the task. There was a great variety of answers. Few of these students were able to explain how they arrived at their solutions, with some students merely copying the equation given in the example.

Grades Eight and Twelve

A third numerical relationship problem was presented to students at Grades 8 and 12.

Sample:

x	y
0	1
1	2
3	$\square$
4	5
6	7

The missing number:

The equation:

Now try this one:

x	y
1	6
2	8
3	10
4	12
.	.
.	.
10	$\square$

The missing number:

Explain how you found the missing number:

The equation

This problem presented more difficulty to students for two reasons. Whereas the previous eighth grade problem alerted students to a relationship between x and y by asking them to explain **what they did to x to get y**, this question gave no hint that the two were related. It required students merely to "explain." Secondly, the number for each variable was presented in order (e.g., 1,2,3,4..and 6,8,10,12..). This prompted students to look at them as two separate sequences with no relationship to one another. (See Appendix.)

Fifty-two percent of eighth graders and 35 percent of twelfth graders gave an incorrect solution and 10 percent at both grades did not attempt the question. Most of the incorrect responses focused on the columns, treating the numbers

as sequences. When asked to supply the equation, these students merely ignored the functional relationship between the sets and wrote a number sentence relating the specific pair. **The pattern is x adds one each time, y adds 2,** wrote one Grade 8 student. (Equation given: $y = x + 14$.)

Thirty-eight percent of eighth graders and fifty-five percent of twelfth graders supplied the correct missing number (24). However, although there was an increase in success rate between the two grades, the percentage of students who were able to *explain* the operation or supply an equation is disappointing. Only 21 percent of eighth graders and 34 percent of twelfth graders were able to explain their reasons for choosing the correct number. Only 13 percent of eighth graders were able to give the equation which describes the relationship. Although this number rose to 28 percent at the twelfth grade level, it is disappointing, given the number of twelfth graders who have taken basic algebra.

a	b
1	6
2	8
3	10
4	12

Which rule fits this table?

A. b = (3 x a) + 3
B. b = (2 x a) ÷ 3
C. b = (3 x a) + 2
D. b = (2 x a) + 4

Percentage of correct responses:

Gr 8	Gr 12		
18%	12%		A. b = (3 x a) + 3
20	10		B. b = (2 x a) + 3
22	16		C. b = (3 x a) + 2
38	61	*	D. b = (2 x a) + 4

Although students were more successful when given a set of equations to choose from (see multiple-choice above), the results in response to the open-ended version suggest the difficulties that students have in generating equations or using algebraic notation as a shorthand for description.

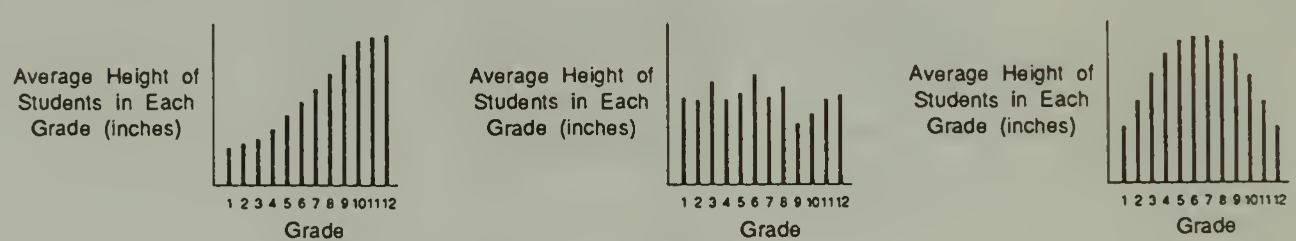
Graphical Relationships

The recognition of pattern and relationship is not confined to numbers. It underlies the understanding of graphical material as well.

Although students perform relatively well when asked to read data from bar graphs, they are not as successful at interpreting graphical display. Response to the science open-ended tasks indicated that students experience great difficulty in depicting relationships between variables. (See **On Their Own: Student Response to Open-Ended Questions in Science.**) The questions below do not involve the mechanics of graphing *per se* (scales, intervals, etc.). Instead they addressed some basic concepts by asking, do students understand graphs as the representation of a relationship between variables?

Grade Four

1. A school has twelve grades in it. A student computes the average height of the students in each grade and makes a graph showing the results. Circle the graph below that would look most like the student's graph.



2. Explain why you chose the graph you did in question 1.

Most fourth graders showed a good understanding of what the graphs were intended to convey. Eighty percent of the students selected the first (correct) graph. Sixty-six percent gave a convincing reason. "As you get older you get taller, and in the first graph the students get bigger and bigger," wrote one student.

"The people in the second graph are all mixed up," another responded. "I don't think the first graders are taller than 9th and 10th graders. In the third

graph, it's mixed up too. 1st graders are not the same height as 12th graders and 2nd graders are taller than 10th graders? I don't think so."

Fifteen percent chose graph 2. Approximately half of those children misunderstood the meaning of the horizontal axis, believing that each child was represented by a number: "Because it's not in order. Not all 12th graders are tall."

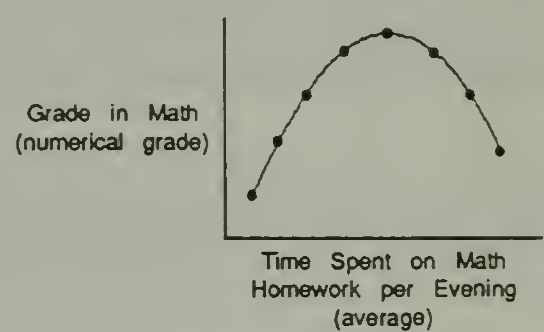
Others based their decision on its general appearance: "The others look too much like a made-up school because they have patterns." The 4 percent of the students who chose the third graph were typically unable to explain their choice.

Judging by their responses to this question, it seems that the large majority of fourth graders were able to transfer their experience with bar charts of frequencies or amounts to a more sophisticated context (i.e., average height), as well as recognizing the relationship between age and height.

Grades Eight and Twelve

A parallel question at the eighth and twelfth grades asked students to interpret data presented in a line graph. The results show: 1) a general increase in success with experience; 2) a gender difference that is associated with age; and 3) the influence of preconception on the interpretation of graphs.

1. A researcher asked many students two questions: "What was your grade on your last math exam?" and "How many hours per night (to the nearest half hour) did you usually spend on math homework?" The researcher than sorted students into groups according to how much time they spend on homework. Finally, the researcher computed an average math grade for each of these groups, and plotted the averages in the graph below.



A quarter of the eighth grade students were able to supply a complete explanation of both negative and positive relationship. Another quarter, undoubtedly influenced by their preconceptions about the effect of studying, could only account for the positive relationship. As one student wrote: "It goes from the low grades to the high grades back to the low grades. The low grades are also the students who spend less time on their homework."

The other half of the students were unable to interpret the graph. Some were confused by the lack of scale: "This graph does not have enough information such as how many hours? Does it increase by 5, 1, 1/2? Therefore I cannot answer the question correctly." Others focused on only one or the other dimension without understanding that the graph described a relationship between variables: "Some scores started low and went up and then went down again." and "The time spent goes up and then goes down." As in the case of the fourth graders, some students believed that each interval represented a discrete score. "Because different people had different answers."

At the twelfth grade level, more students were able to account for the curvilinear shape of the slope. Nevertheless, the incorrect responses followed the pattern of the eighth graders. (See Table below and Appendix.)

Girls generally experienced more difficulty with this item than boys, and this gender difference increased with age. Whereas 6 percent more boys than girls correctly interpreted the graph at grade eight, there was a 19 percent difference at grade twelve. Boys who understood the graph gave a complete explanation, whereas girls tended to explain only the positive relationship between grades and homework time or did not address the relationship at all.

Graph of Study Time and Grades

	Gr 8		Gr 12	
	M	F	M	F
Complete explanation (accounts for both positive and negative slopes)	27	21	49	30
Partial explanation (accounts for positive or negative slope)	31	24	25	34
Does not address relationship	35	46	13	27
Blank/I don't know	7	9	11	8

Prior knowledge or preconceptions seemed to influence students' interpretation of the graph. Whereas younger students found it difficult to believe that homework and grades were not positively related (and, consequently, tended to ignore the negative relationship that was portrayed), older students talked about fatigue and ability. It may be that experience with the content accounted for the increased success at grade 12.

Grades Eight and Twelve

In another set of items, students were asked to construct their own visual relationship. Because we were interested in their understanding of functional relationship, we did not ask them to consider scales, the ordering of values, and the identification of coordinates. The results of the 1986 open-ended question requiring them to draw a graph suggested that students have great difficulty with these aspects of visual representation.

1. Two of the three graphs below have not been completed. Draw lines in the last two so that the graphs look the way they would if you had real numbers to use. (Hint: Look at the labels along the axes first. Then pretend there are numbers along them. Think about how the two quantities are related. Use the first graph as an example.)

2. Explain why you drew the graphs the way you did.

Students responses to the time/age graph differed according to their perspective. Consequently, graphs were judged by the extent to which they reflected the explanation given. For example, when students compared themselves with much older people and stated: "As you get older, the more time you need to run a mile," a straight linear relationship was considered correct. On the other hand, when a student wrote of the changing pattern from childhood to old age, a curvilinear slope was required.

Even accounting for the flexibility in scoring, this proved to be a difficult task for all students. (See Appendix for examples.) Only 29 percent of eighth graders and 34 percent of twelfth graders were able to draw slopes that reflected their statements.

Many of the incorrect responses were visual in character, particularly among twelfth grade girls. These students appeared to think in terms of an increase

in speed rather than a decrease in time. Their explanations described age as related to "running fast" and, consequently, they tended to draw positive slopes. Approximately 31 percent of twelfth grade girls fell into this category, in contrast to 11 percent of the boys. When boys drew an incorrect slope, their explanation showed a more general lack of understanding. The example below illustrates a typical response. It also reflects the tendency among students to start the slope at the intercept of the axes, regardless of how the relationship is described.

In the second graph, very young children, especially babies, are not very quick runners usually. When people start to get older, then into their prime, their legs get more mature to run faster. As they get older they get weaker bones and muscles, so they can't run as fast.

A new car can sell for the most because it is unused and in perfect condition. As the car gets older it gets more worn down, goes more miles, gets more dents and scratches, etc. so the price decreases.

The third graph, which concerned the relationship between a car's age and its value, was perhaps of greater interest and familiarity to the students. It had a much higher success rate, and the differences between boys and girls were also less apparent. This may be due to the fact that, in this item, the verbal and visual information reinforce each other, as a decline in values suggests a descending slope. However, some students recognized that the slope can be curvilinear. The following example illustrates the value of requiring an explanation to such items; otherwise, who would know that the student was referring to antique cars in her graph?

2nd graph I did—The price of cars go down, until they're antique. The prices then rise.

Geometry and Measurement

Geometry: Plane Figures

Students' responses to multiple-choice questions concerning geometry have been described in detail in a separate publication, **Moving Geometry from the Back of the Book** (Department of Education, 1987). In the open-ended tests, we examined fourth graders' conception of a simple plane figure by asking them to describe a rectangle.

> Describe a rectangle in words for a person who does not know what a rectangle is. Describe it completely so that the person will be able to tell if any figure is or is not a rectangle.

While 3 percent of the students were able to give a complete, defining description of a rectangle, approximately half gave an adequate informal description. (See Appendix for complete results.) These informal descriptions included the notion of four sides and pairs of equal sides. As expected, students' descriptions tended to stress the prototypical long rectangle and they showed the usual tendency of differentiating between rectangle and square. Generally, few students mentioned angles, although 9 percent used the word "corners" to describe right angles. A typical response was:

> A rectangle is a figure with four sides. Two sides are long and two sides are short. The rectangle is two squares put together. A rectangle has four corners.

Other students referred to common objects to help them in their description, such as, "It looks like a stick of gum." However, many were unclear about the figure. (See Appendix.) For example, 13 percent described a triangle. One very graphic description:

> A rectangle is two slanted lines with a straight line at the bottom and it is shaped like an A but the line in the middle gets taken away and put at the bottom of the A.

Still others referred to a solid figure, while retaining the concept of length:

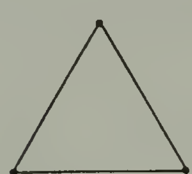
> A rectangle is like a cube but it is larger than a cube and it has 6 sides and it has no curved surfaces, but it does have flat surfaces and it has six vertexes and it is longer than a cube too.
>
> A rectangle has 4 sides. It is shaped like a tissue box. It is not shaped like a cube.

The fact that 50 percent of the students did not give an adequate description of a rectangle indicates how fragile their understanding of plane figures may be. When asked to recognize prototypical figures as on the multiple-choice questions, students tend to perform well. For example, when presented with a set of standard plane figures, over 90 percent of third graders correctly identified rectangles, squares, and triangles. However, the multiple-choice question may give us an inflated impression of what students know. Description demands a greater degree of understanding. In order to describe a figure, students must consider its properties, those attributes that make it what it is. This is active knowledge, which many children seem to lack.

Geometry and Measurement: Area

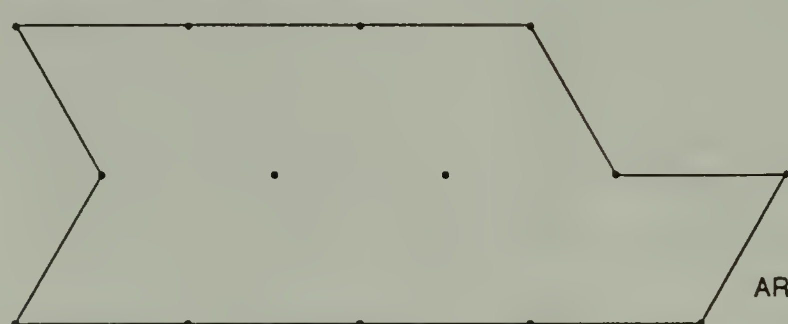
An understanding of the relationship between geometry and measurement has important practical uses, as well as being crucial to success in more advanced courses in calculus. The questions below focus on the relationship between geometry and measurement, asking students to express the area of a figure in terms of a non-traditional unit. Here, we were particularly interested in students' ability to imagine how a figure might be partitioned – a necessary condition for calculating the area of irregular shapes.

Grade Four



IS ONE UNIT OF AREA.

What is the area of the figure below?

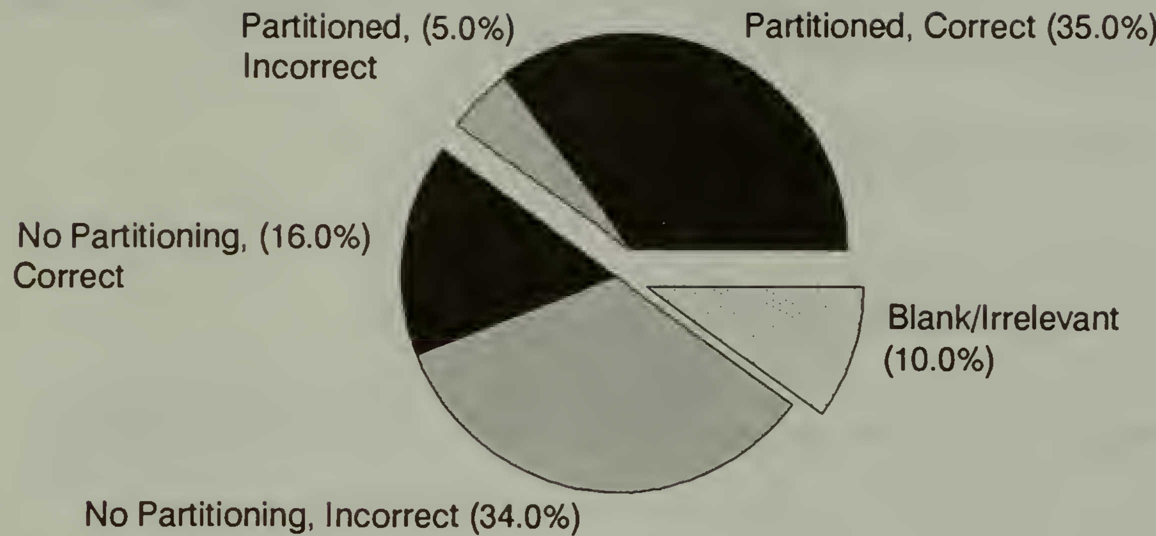


AREA: ____________ units

The most striking feature about the responses to this item is the effect of partitioning on the accuracy of response.

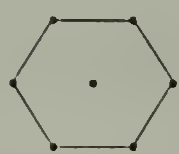
Over 80 percent of the students who partitioned were able to calculate the correct answer (14 units), while fewer than 30 percent of those who did not partition were successful. A slightly disappointing note was the number of students who erased their partitions, presumably after they had counted the units. Evidently, they believed that their method might be considered cheating or was somehow unacceptable.

Fourth Grade



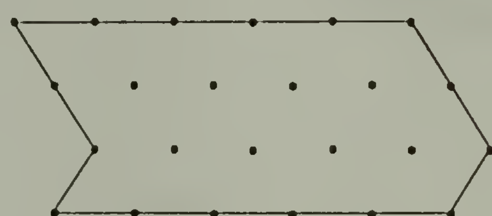
Grades Eight and Twelve

At the eighth and twelfth grade levels, the measuring unit was more complex, although the question was essentially the same.



IS ONE UNIT OF AREA.

What is the area of the figure below?



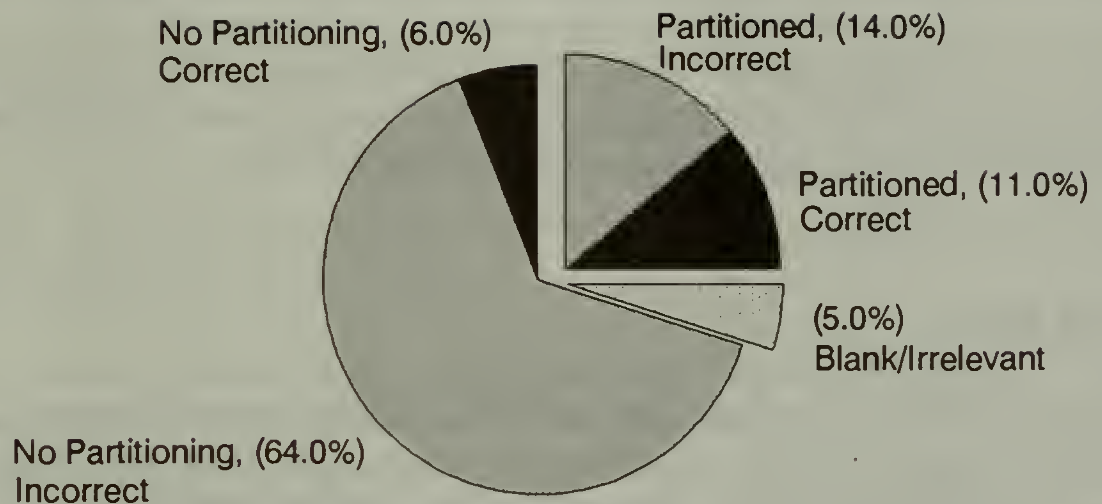
AREA: ________________ units

As a result of either the unit itself or a lack of recent experience, only a quarter of eighth graders attempted to partition the figure. Eighty percent of the eighth graders did not partition, with few giving the correct answer of 5 units. The most popular answer was 8, which could have been obtained either by counting the number of whole, hexagonal units which could be placed in overlapping fashion in the figure or by counting the number of interior points. Others seemed to count all the points (24). Obviously, these students did not have the concept of area as a covering unit. (See Appendix for complete list of responses.)

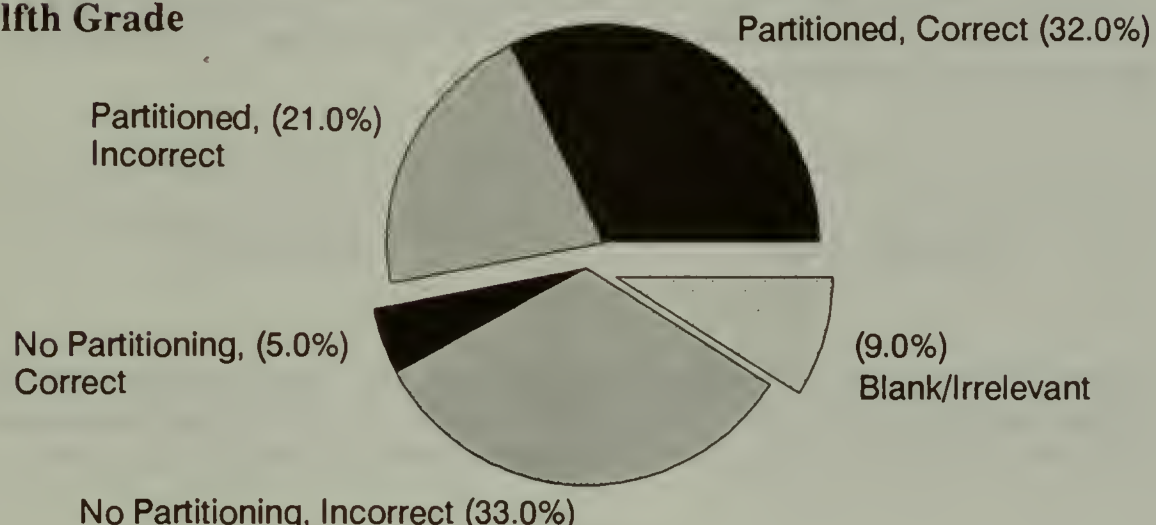
The effect of partitioning on success is clearly seen at Grade 12. Both partitioning and success rate increased at the older level. Over half of the twelfth graders partitioned, and most of those obtained the correct answer. As was the case with Grade 8, most of the students who did not partition counted the interior points.

Of the students who did partition but obtained an incorrect answer, some counted only intact hexagons, not recognizing that the hexagons themselves can be partitioned into congruent triangles.

Eighth Grade



Twelfth Grade



From the analysis above, it appears that the open-ended item gives us a better indication of students' conception of area than multiple-choice items in which they are called upon only to supply the correct answer. In these cases, students more often choose the perimeter than the area.

Consequently, the argument is made that students difficulty with the concept of area lies in vocabulary, i.e., that they confuse *perimeter* with *area*. These results suggest a more fundamental problem. Students do not understand area in terms of a covering unit, nor do they think of a plane figure as capable of being dissected into its component parts. Their difficulties are both conceptual and perceptual.

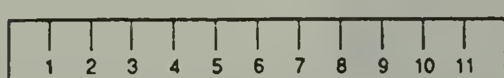
Measurement: Precision and Accuracy

Judgment is an important element in measuring. In addition to the more mechanical procedures, students have to be able to decide what degree of accuracy is necessary, given the requirements of the task.

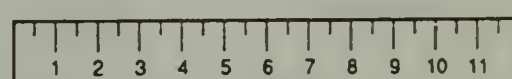
Grade Four

In order to explore such questions as "Do students recognize the extent to which instruments differ in precision? Do they understand the relationship between the precision of the instrument and the task involved?" we asked the following question of Grade 4 students.

1. Christine and Carla each owned a chair with a broken leg. They decided to replace their broken legs. They both measured the length of the good legs on their chairs to find out how long to make their new legs.



Christine's Ruler



Carla's Ruler

Christine measure things to the <u>nearest inch</u> using her ruler. Carla measures things to the <u>nearest half inch</u> using her ruler. If the two girls decide to repair chair legs as a hobby using the rulers shown above, who will probably be more successful? Explain why.

Two-thirds of the students believed that Carla would produce the best results. Most recognized the need for precision and reasoned that her measurement would be closer to the actual length of a leg.

> I think Carla would be more successful. I think so because Christine's ruler is by the inch, and Carla's ruler is by the half. So Carla's ruler would be closer to the exact measurement.
>
> Carla will be more successful because the length will probably not be an exact inch so she would probably have about the right length for the leg.
>
> Carla would because if the chair leg was 13 ¼ inches long 3 ½ would be closer than 13 or 14.

Twenty-nine percent believed that Christine would be more successful. It was clear from the explanations given that most of these children had little understanding of the relative merits of precision and accuracy. They appeared to look at the task and reason that measuring to a half-inch was more complex; therefore, more difficult; therefore, there would be less chance of accuracy. This confusion may have come from their own experiences. Some typical responses are given below:

> Christine, because it will take her less time and work, because if you measure with inches, it will measure easier.

> Christine would probably be more successful. People usually measure thing by the nearest inch and you can get things by the inch much easier.

Numerical and Statistical Concepts

Estimation and Reasonableness

The importance of estimation is often ignored. We may think of it merely as a short-cut for calculation or a good guess. However, students' ability to estimate can give us a good idea of how well they understand a system of measurement (e.g., decimal, linear, volume) and what they consider a reasonable degree of accuracy.

Grade Four

At fourth grade, we concentrated on numerical estimation in the following question:

> Explain in words how you would estimate 29 X 310. Then tell what your estimate would be.

Sixty-five percent of the students showed an understanding of what is meant by estimation in this context. Approximately half rounded 29 to 30 and 310 to 300, while another 15 percent rounded one or the other numbers in the pair.

Grades Eight and Twelve

In these grades, we included estimation tasks in both numerical and verbal contexts. The numerical question was similar to that given at the younger level but involved decimal fractions.

> Explain in words how you would estimate 88.74 X 105 in your head. Then tell what your estimate would be.

In this case, perhaps the simplest method would be to round 105 to 100. This was the course followed by 37 percent of eighth graders, with 26 percent allowing for some sort of compensation by rounding 88.74 up to 89 or 90. Others rounded 105 up to 110 (8 percent of students) or used the number as given and rounded 88.74 to a whole number (19 percent). However, a sizeable proportion of students were baffled by the task. Twenty percent left the question blank, and 17 percent completed the multiplication with the numbers unchanged, giving a description of their multiplication procedure. Although a few rounded their final answer to the nearest whole number, these students obviously did not understand the purpose of estimation.

In contrast to the performance of fourth graders, the results of these older students was disappointing, particularly at the twelfth grade. Not only was there little growth in success, but it could be argued that eighth grade students showed a better understanding of the task than the older students. More twelfth graders appeared to perceive a need for an absolutely correct answer rather than an estimate.

Grades Eight and Twelve

Most estimation activities take place in a context. In fact, it is the context that determines much of the decision-making in estimation—the degree of precision, the type of instrument to use. The practical question given below deals with numerical estimation, but puts it into context.

> There are 310 eighth graders at Rockmont Junior High. A testing company is sending 310 tests to the school to be administered to the eighth graders in their homeroom classrooms. The company has to estimate how many direction manuals to send. There should be one for each classroom. Approximately how many manuals should be sent? Explain how you came up with your estimate.

Approximately half the eighth graders and 70 percent of twelfth graders replied that between 10 and 20 manuals were needed. In their explanations, many students indicated that they used their own experiences to estimate a reasonable class size. However, some students (15 percent at Grade 8 and 10 percent at Grade 12) were puzzled enough by the question to leave it blank, while 5-7 percent replied that the necessary information was missing. A sizeable proportion of eighth grade students (20 percent) focused only on the numbers themselves, without noting what they represented.

Some typical responses were:

> 39 manuals. I divided 8 into 310.
>
> 5, because 5 is the only number that goes into 310 evenly.
>
> 38, because I divided 310 by 8.

This type of response—an unthinking manipulation of numbers—is also common to word problems on multiple-choice questions. In the case of the open-ended exercise, the requirement that they explain their method may have alerted them to examine the elements of the problem rather than to plunge ahead with some sort of calculation.

Statistical Concepts: Reasoning about an Average

Although multiple-choice questions can be used to measure students' ability to evaluate arguments or supply missing elements or even identify valid conclusions, their ability to generate arguments can only be measured by asking them to do it – either orally or in writing. In this case, we asked them to construct an argument concerning the effect of adding a constant to the numbers in a set. Because of its familiarity, we used the concept of average test score to determine how well students were able to use mathematical language to argue convincingly.

> Roger doesn't believe that adding a constant (the same number) to every student's test score will simply change the average test score by that same amount. Use the space below to convince Roger that this is

It was obvious that students were unfamiliar with this kind of question. Twenty percent of eighth graders and a quarter of twelfth graders did not even attempt to answer the question. Of those who responded, fewer than a quarter of the students at Grade 8 were able to give a convincing argument using a worked example (i.e., compute two averages, with the scores in the second set higher by a constant). At Grade 12, this figure rose to 34 percent, while another 4 percent were able to give a correct narrative explanation for the impact of a constant on the sum of the scores. Very few students gave an algebraic proof or an appropriate logical argument in mathematical terms such as the one below given by a twelfth grade student. (See Appendix for breakdown.)

> Roger doesn't believe that adding a constant (the same number) to every student's test score will simply change the average test score by that same amount. Use the space below to convince Roger that this is true.

Let x = constant

Let 4 = number of tests ~~[illegible]~~

a, b, c, d = individual test grades

$\frac{a+b+c+d}{4}$ = average without added constant

$$\frac{(a+x)+(b+x)+(c+x)+(d+x)}{4} = \frac{a+b+c+d}{4} + \frac{4x}{4} = \frac{a+b+c+d}{4} + x$$

average grade with added constant

A necessary condition for a convincing argument in this task was an understanding of the concept of average. Here, many students failed before they started. For example, the students whose work are shown below appeared to interpret "the average test score" as the relative ranking of test scores.

> Roger doesn't believe that adding a constant (the same number) to every student's test score will simply change the average test score by that same amount. Use the space below to convince Roger that this is true.

Someone could receive a 73 on a test. Another could get a 85 on a test. If you add 5 to each they still be different!

① 73 - C− ② 85 B

$\begin{array}{r} 73 \\ +\ 5 \\ \hline 78 \end{array}$ C+ $\begin{array}{r} 85 \\ +\ 5 \\ \hline 90 \end{array}$ A-

73 = C−
Changed TO:
78 = C+

85 = B
Changed TO:
90 = A-

Roger doesn't believe that adding a constant (the same number) to every student's test score will simply change the average test score by that same amount. Use the space below to convince Roger that this is true.

	Pam	+	constant	=	result	
D	60		10		70	C
	Sandy					
C	75		10	=	85	B
	Dave					
B-	82		10	=	95	A
	Roger					
A	94		10	=	104	A++

Even among those students who were able to illustrate the effect of the constant, few recognized that only two numbers are needed to obtain an average. Most used four or more in each set. Furthermore, approximately half the students used numbers that were difficult to compute, such as 87, 79, and 93. These students may have believed that, in order to be convincing, the numbers themselves had to be authentic. They did not understand that it is the quality of the argument rather than the complexity and abundance of the numbers themselves that is critical.

Roger doesn't believe that adding a constant (the same number) to every student's test score will simply change the average test score by that same amount. Use the space below to convince Roger that this is true.

original test scores:	scores + constant (10):
280	290
83	93
87	97
83	93
90	100
77	87
94	104
7)594.0000	7)664.0000
average: 84.857	average: 94.857

As a by-product of this strategy, 10 percent of twelfth graders and 15 percent of eighth graders were unable to perform the correct computation. In addition, however, there appears to be an underlying misconception of what qualifies as a convincing argument in mathematics. Many students seemed to believe that if the text is lucid enough, it will be convincing. Such students personalized their responses, restated the question, or relied heavily on the use of the vernacular and persuasive language in their presentation. They were distracted by the context and seemed unable to look at the generalized mathematical structure involved.

Although, by Grade 12, a few more students were correct, more used easy numbers, and more were able to use their knowledge of algebra to make a convincing argument, the achievement level was poor, in view of the fact that the context and substance of the question should be familiar to all students by this age. The majority appeared to be baffled by the task.

Summary of Results and Implications for Instruction

Variables and Relations: Numerical Patterns

Finding patterns is the essence of mathematics. It is disappointing, therefore, that students did not cope more adequately with the tasks in this category or did not become more proficient as they got older. On the contrary, fourth graders were considerably more successful with this type of question than eighth and twelfth graders. The results suggest that this kind of task may be more unfamiliar to older than to younger students. This is an ironic conclusion, given the dominance of algebra in the high school curriculum. Certainly, the requirement that they explain their reasoning proved to be very difficult for most older students. However, it gave us valuable insight into students' understanding—insight that could not have been obtained if we had been satisfied merely with a correct response. From their explanations it is clear that many students arrived at what seemed to be a correct solution merely by chance or by focusing on a single instance. Most were also at loss to give an equation which would describe the functional relationship between the two sets of numbers. Again, generating an equation (as opposed to recognizing one) proved to be difficult for almost all eighth graders and for nearly three-quarters of twelfth graders. Even those who were able to produce the correct solution could not look beyond the specific case to give a generalized rule about the two sets of numbers. Formulating an equation demands an understanding that algebraic equations are nothing more than a symbolic way of describing an existing relationship between numbers. The difficulty that students had suggests that teachers make a greater attempt to provide informal activities where students create or recognize patterns and generate their own algebraic sentences to describe them.

Variables and Relationships: Relationships in Graphs

Again, fourth graders appeared to have a better sense of what graphs are meant to portray than older students. It may be that, in later grades, so much emphasis is placed on the mechanics of graphing that students forget what graphs are essentially designed to portray—a relationship between variables. Whatever the cause, a disappointingly small number of eighth grade and twelfth grade students were able to interpret or draw line graphs that described relationships.

The argument has been made that line graphs are inherently difficult, relying upon an ability to abstract, as well as upon proficiency in numerical ability. On the contrary, we believe that line graphs are too useful a teaching device to wait until the student is "ready." This type of graphing should not be limited to those students who have shown themselves to be adept at numbers.

Students so seldom have the opportunity in mathematics to describe relationships without the explicit use of numbers that, when asked to plot a linear equation, they often don't understand what they are representing. They don't realize that they are portraying the functional relationship between variables. However, the problems contained here allow students to do something with quantities other than compute.

By using simple graphs, quickly drawn, without scale or finely determined coordinates, students can focus on the slopes themselves and the ideas that they are meant to convey. How are different variables related to one another? How often are they linear? Which variables have similar relationships? Which have inverse relationships? These kinds of questions can make mathematics more real to students, as well as developing a more intuitive understanding of later work in algebra and calculus.

Geometry and Measurement

It is well known that little time is devoted to geometry throughout the grades. It is not surprising, therefore, that students don't have a firm understanding of the properties of a figure or how different figures relate to one another. On the other hand, despite the fact that considerable time is spent on measurement topics such as area, growth in understanding is slow

and shaky. Teachers contend that the problem is one of vocabulary – students confuse the words *perimeter* and *area*. Undoubtedly, this confusion accounts for some errors, but responses point to conceptual and perceptual reasons as well. Students do not understand the relationship between the properties of figures and their measurement. Perceptually, they fail to see how simple regular figures can be inscribed in curved or irregular ones, or how complex figures can be partitioned into simpler ones. Conceptually, they don't recognize that area is the measurement of surface and therefore is dependent upon shape.

The most prevalent way of presenting area, through regular quadrilaterals, puts too much and too early a focus on the simplistic application of formula to the detriment of understanding. More stress on irregular figures and non-standard units would highlight the necessity for looking at the figure as enclosed space. A simple formula, which should be regarded merely as a computational short-cut, too often becomes the only tool that students can use to solve problems.

Measurement: Precision and Accuracy

An understanding of the relative need for precision and accuracy is also a matter of judgment, and the role of context is an important consideration. Approximately two-thirds of the fourth grade students agreed that a more precise measurement would lead to better results when attempting to balance chair legs. The remainder appeared to be more concerned with accuracy, believing that a more precise measurement would be more difficult to carry out and, therefore, stood less chance of being "correct."

Numerical Concepts: Estimation and Reasonableness

Estimation is not the same as "rounding numbers," although many students appear to think that this is the case. Estimation requires both a proficiency with a system of measurement (whether it be decimal, linear, or some other) and a degree of common sense, what some would call "reasonableness." Approximately 65 percent of students at all three grade

levels understood the meaning of the word and were proficient enough with the decimal system to successfully carry out an estimation.

When presented with a problem that might come up outside the classroom, eighth graders found the task more difficult. Half of them could not apply their everyday experience (their knowledge of the number of students in a class) to this "math" problem, and many retreated to a stimulus-response reaction of manipulating the numbers given. Probably reflecting greater maturity, more twelfth graders (70 percent) were able to respond reasonably, but many remained baffled. Their answers are in sharp contrast to what is known about the "expert" approach to problem solving, whereby qualitative aspects of the problem are of primary consideration, to be followed by numerical calculations.

Statistical Concepts: Reasoning about an Average

It is important to reason *with* numbers, such as in the practical problems discussed above. It is also important that students be able to reason *about* numbers. Unless they firmly believe that mathematics is a reasonable system, students will be reduced to accepting and memorizing what teacher or textbook says.

What was designed to be a measure of reasoning ability acted instead as a measure of students understanding of the concept of an average. It was obvious from their discussion that many students had only a shaky understanding of what an average means. Many more had little understanding of the hallmarks of a mathematical argument. For most students, a convincing argument was one that was specific and personal, replete with realistic examples. That so many students (from one-fifth to one-quarter) did not even attempt the question suggests that they have had little experience is discussing either mathematical concepts or some of its basic applications. Since logical reasoning is the hallmark of mathematics, it is particularly discouraging that only 4 percent of twelfth graders could give either a correct narrative explanation or algebraic proof of an average.

Conclusions

The questions given in the open-ended section of the assessment yielded valuable information — not only about what students know and understand, but about how they view mathematics itself. That so many students did not even attempt to answer the questions is revealing. Perhaps they did not see the connection between the questions that were presented and what they have come to believe mathematics represents — not a logical system of relationships, but computation. In this case, students may have thought that they had nothing to contribute. When asked to apply basic concepts in an unfamiliar context, they were baffled.

However, we believe that the value of this type of testing reaches beyond the specific results that are reported here. For the teacher, open-ended questioning provides a better insight into students' understanding than those more typical questions that demand only a correct response. Again and again we see that an answer alone is but a weak indicator of understanding. In contrast, when we ask students to explain or to justify their responses, we can evaluate not only the procedures which they used, but the premises upon which those procedures were based. Furthermore, the value of this kind of task extends beyond the context of assessment. Any one of the exercises given in this report could serve as launching point for the kind of discussion that educators say is so important — and so absent — from mathematics instruction today.

Open-ended questioning is valuable for the student as well. When students are asked to puzzle and explain, to apply their knowledge in an unfamiliar context, they must construct meaning for themselves by relating what they know to the problem at hand. In other words, they must act like mathematicians. This kind of activity encourages them in the belief that mathematics is primarily a reasonable enterprise, founded in the relationships apparent in everyday life and accessible to all students, whatever age or level of ability. It is not by accident that the first three standards put forth by the National Council of Teachers of Mathematics focus on **problem solving, communication,** and **reasoning** in the study and teaching of mathematics.

Notes

Postscript

Members of the Mathematics Advisory Committee reviewed the results and made the following comments.

On the importance of graphical exploration:

> It is apparent that students at all three grade levels did not use partitioning, diagrams, pictures, and labels to help them understand and solve the examples. Some of the students who did carefully erased all their work. Teachers should put more emphasis on these appropriate and legitimate techniques for problem-solving.
>
> Christine Thompson
> Fourth grade teacher, Leicester

On the importance of assessing students' understanding:

> Much more can be determined about a student's understanding of a particular concept by knowing how he/she arrived at that answer (whether it is correct or incorrect). This, in turn, should lead to better ways of teaching these concepts to children.
>
> It may force us to examine how children learn (and retain what they learn). We, as teachers, think we are doing such a great job, but we are not always aware of the way children reason. Are we responsible for their faulty learning? It is not easy for a child to explain why he/she did something. We must explain things to them in a meaningful way. I think this presents a good challenge for us.
>
> Jack Waite
> Director of Mathematics,
> Winchester

On instruction:

Everywhere, it seems, the open-ended report reveals incomplete math constructs among all ability levels. I believe that the instruction may be incomplete as well in many instances. We who teach math are probably not taking the time to develop sufficient "enabling skills" for constructs such as area and vector. More emphasis is needed, then, on enabling activities such as covering (area), what's my rule? (functions/algebra), estimating contests (estimation),elementary data analysis (analysis and synthesis skills with graphs), and better development of geometry.

Thomas Risoldi
Director of Mathematics, Salem

On the importance of students' explanations:

The most important finding for me was that schools do not ask students to justify their answers. We frequently ask for an answer, but seldom ask *why*. In addition, students do not have a good mathematical vocabulary. They do not know how to "talk" mathematics. This is obvious when they try to explain their answers.

Paul Lyons
Coordinator of Mathematics,
Cambridge

On open-ended questioning:

The open-ended questions in the statewide tests allow the committee to observe, and thereby report to other educators, the thinking processes of students at various grade levels.

The multiple-choice type questions permit only an interpretation of resultant answers, but not thinking patterns. The educational implications of these thinking patterns, presented in this report, must be studied carefully by all people involved in mathematics education.

Winston Rose
Department Head,
Masconomet Regional
High School

Appendix

Reporting Category: Patterns and Relationships: Numerical
Grade Level: Eight and Twelve

Correct number solution:

Grade 8	38 %
Grade 12	55 %

Type of **explanation** given for a *correct* response (percentage of total sample)

	correct column	correct function	incorrect column	incorrect function	other incorrect
Grade 8	11	10	5	3	9
Grade 12	17	17	5	5	11

Correctness of **equation** given with a *correct* response (percentages of total sample)

	correct	incorrect	blank
Grade 8	13	16	2
Grade 12	28	19	8

Reporting Category: Patterns and Relationships: Graphical
Grade Level: Four

Response Summary	Frequency (%)
a First graph: good reason	66
b. First graph: poor reason	14
c. Second graph	15
d. Third graph	4
e. Blank/ I don't know	3

Examples:

a. Graph 1 because each year you get bigger. In the other ones you get bigger then smaller.

b. Graph 1 because Graph 2 and 3 are uneven and wrong. In graph 2 it shows that 6 inches are taller than 12 inches. In graph 3 it shows that 1 inch is the same size as 12 inches.

c. Because some years you grow a lot more than other years when you grow an inch.

c. Because some 1st graders are bigger than third graders. Some sixth graders are smaller than fourth graders.

c. I chose graph 2 because the students didn't come in order by height, they took it the way they came.

d. Graph 3 because some kids are taller than other kids even if they're in a lower grade.

Graph of Study Time and Grades

Reporting Category: Patterns and Relationships: Graphical
Grade Level: Eight and Twelve

Response Summary	Frequency (%) Grade 8	Grade 12
a. Complete explanation (accounts for both positive and negative slopes)	24	38
b. Partial explanation (accounts for only positive or negative slope)	27	31
c. Does not understand meaning of graph	41	21
d. Blank/I don't know	8	9

Examples of categories (Grade 8):

a. You need to spend time on your homework, but too much time may confuse you. That is why in the middle it is the highest and on the ends it is the lowest on the grades in math.

a. People who didn't study very much or not at all didn't do very well. People who studied until they knew it got the top scores. People who studied until they were exhausted or too much didn't do as well because they could have confused things.

b. Because the less time they spent on homework each evening, the lower the grade. The more time they spent, the higher the grade.

c. It's just the way that it came out after he connected the dots that showed these test grades according to the average time they spent on homework.

c. The graph has this shape because in the beginning the students did less homework, so their grades were poor. Then they studied more and got much higher grades, until they thought their grades were good enough. They didn't have to study much anymore and their grades went down once again.

c. It is that way because on one night that student had studied more than other nights.

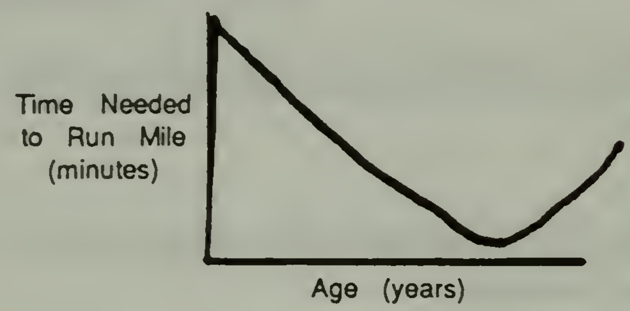
Time/Age Graphs

Reporting Category: Patterns and Relationships: Graphical
Grade Level: Eight and Twelve

Response Summary	Frequency (%) Grade 8 M	Grade 8 F	Grade 12 M	Grade 12 F
a. Acknowledges changing pattern	18	15	33	23
b. Negative slope: good explanation	15	10	5	9
c. Positive slope: good explanation	16	14	11	31
d. Explanation differs from graph drawn	26	22	35	16
e. Other incorrect or irrelevant slopes and descriptions	10	26	4	6
f. Blank/ I don't know	13	12	13	15

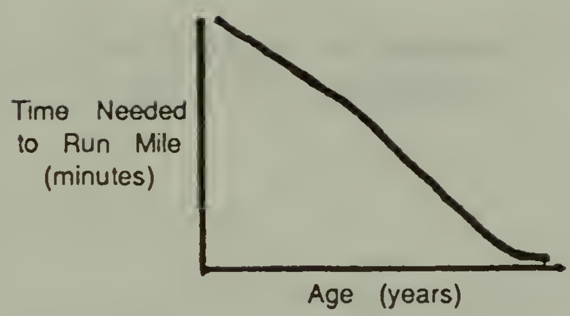
Examples (Grade 8)

a.



When you are young you don't go as fast, but when you get older you get tired easily.

b.

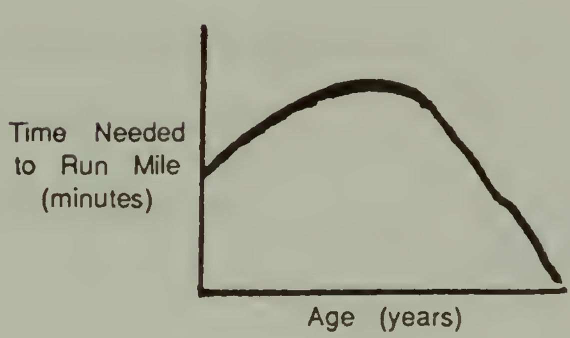


The older the age was, the faster the person would run.

c.

The older you get the more time you need to run.

d.

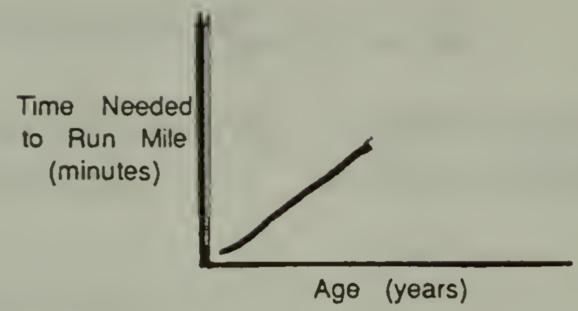


When you are real young you can't run that fast, but as you get older, you get better. But after awhile your speed slows down as you go up in age.

e.

The older you are, you need to run more to make you healthy.

e.



It would depend on the age and if you were in shape to do the mile.

Reporting Category: Geometry and Measurement: Plane Figures
Grade Level: Four

Response Summary	Frequency (%)
a. Correct, complete, defining description of rectangle	3
b.* Correct, informal description of rectangle (e.g., "corners" for angles)	9
c. 4 sides and pairs of equal sides (no mention of angles)	25
d. Pairs of equal sides, horizontal longer than vertical	25
e. Other description	8
f. Refers to appropriate real objects (e.g., dollar bill)	5
g. Refers to rectangular solid	3
h. Describes triangle	13
i. Blank, irrelevant	9

* Definitions categorized as correct do not have to describe all characteristics of a rectangle. For example, "4 sides, opposite sides parallel, one right angle" must be a rectangle even though no mention is made of the opposite sides being equal in length. All figures described are assumed to be closed.

Estimating Need for Manuals

Reporting Category: Numerical Concepts: Estimation and Reasonableness

Grade Level: Eight and Twelve

Response Summary	Frequency (%) Grade 8	Grade 12
a. 10-20 manuals	54	71
b. Explanation describes appropriate method: calculation error	6	6
c. "Not enough information"	5	7
d. Other	20	6
e. Blank/ I don't know	15	10

Examples (Grade 8):

a. 12. You say about 25 in each classroom, so you divide 310 by 25 and whatever your answer is, that is how many booklets.

a. 15. There is probably about 20 students in each class. Divide 20 into how many students. This will give you how many homerooms there are.

b. Approximately 104 because there are approximately 30 students in each class, so you divide 310 by 30 and round off, just in case there are more students per classroom. (Student did not recognize need for decimal in division.)

c. You have to know how many homerooms there are.

d. Approximately 5. There are two junior high grades. So divide, then estimate that there are about 31 kids in a class. (310/2 = 155. 150/30 = 5)

d. There are about ten students per homeroom. So therefore you would have to send around 31 manuals.

d. 5. If there were 62 students in one classroom then 5 manuals were needed.

d. 74 test manuals. Divide 310 by 4 homerooms.

d. I would assume 4 because that is the number of majors (math, English, science, social studies) and homerooms are usually located in the 4 major subject classrooms.

Reasoning about an Average

Reporting Category: Statistical Concepts
Grade Level: Eight and Twelve

Response Summary	Frequency (%) Grade 8	Grade 12
a. Correct narrative explanation or algebraic proof	0	4
b. Computes 2 averages, second is higher by a constant	22	34
c. Adds constant to score, does not compute	6	4
d. Computational error in attempt to average	9	6
e. Incorrect or unconvincing narrative	18	15
f. Irrelevant, misunderstands question	26	15
g. Blank	20	24

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Teachers and coordinators who have been involved in the scoring of the open-ended questions have found it to be a worthwhile and interesting professional experience. This year, the reading and scoring of the 1990 open-ended questions will take place at a two-day session during the week of June 25-29. If you would like to participate, please fill in and mail this application and you will be sent further details.

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I would be interested in scoring the following grades and subject areas:

GRADE LEVEL:

- ☐ Fourth
- ☐ Eighth
- ☐ Twelfth

SUBJECT AREA:

- ☐ Reading
- ☐ Mathematics
- ☐ Science
- ☐ Social Studies

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